

Nucleon mass splitting in the isospin medium

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ABSTRACT

Using the AdS/CFT correspondence, we investigate a nucleon mass splitting and nucleon-pion coupling in the isospin medium. We find that there exists a nucleon mass splitting which is exactly given by the half of the meson mass splitting because nucleon has the half isospin charge of the charged mesons. In addition, we also investigate the nucleon-pion coupling, which requires the modification of the known Abelian-type unitary gauge fixing term because non-Abelian fluctuations should be taken into account in the isospin medium. In this paper, after constructing an appropriate unitary gauge fixing term, we find that in spite of the nucleon's and meson's mass splittings, there is no nucleon-pion coupling splitting in the isospin medium.

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1 Introduction

Understanding strongly interacting systems is one of big issues in physics. However, there is no traditional systematic tool to explain such phenomena. Recently, there was a new challenging idea to understand strongly interacting systems like the low energy QCD and condensed matter system by using the holographic method, the so called AdS/CFT correspondence [1, 2, 3, 4]. In this paper, we will apply the holographic method to the low energy QCD in the isospin medium and investigate the nucleon mass spectra and their coupling with pion. In order to understand physics in a certain medium, in general, it is important to know the interaction between the medium and fundamental excitations. In the low energy QCD, the nuclear medium is strongly interacting system, so there is no way to explain its physics with a traditional field theory technique. To understand such a system by using the AdS/CFT correspondence, one first knows the corresponding dual gravity theory and its geometric solution. There are several known geometric solutions. The thermal AdS geometry with a hard or soft wall is dual to the confining phase of the pure gauge theory while the Schwarzschild AdS black brane is mapped to the deconfining phase at finite temperature [5]-[13]. These works were further generalized to the gauge theory with matter [14]-[26]. For example, the Reissner-Nordström AdS black brane can describe a quark medium at finite temperature and its zero temperature version, the so called thermal charged AdS geometry with a hard wall, is dual to the nuclear matter due to the confining behavior. There is another interesting geometry representing the isospin medium, which has only the isospin chemical potential but not the isospin number density [27]-[31]. Due to its simplicity and analyticity, studying the isospin medium seems to be helpful in understanding the medium effect of the holographic QCD and condensed matter system.

There were many interesting works related to the holographic nucleon masses and nucleon-meson coupling in the AdS space dual to the pure Yang-Mills theory [32]-[39]. In this case, since there is no isospin interaction, the isospin charges of the excitations are not important. In other words, it is sufficient to turn only on the Abelian fluctuations without regarding the isospin structure. This is not true anymore in the isospin medium. Since the isospin medium has a nontrivial isospin chemical potential, charged excitations usually interacts with the isospin medium which can change some physical properties of the excitations. For example, in the holographic pure Yang-Mills theory dual to the AdS space proton and neutron are indistinguishable because of the absence of the isospin interaction. So their masses become degenerate. On the contrary, charged mesons and nucleons in the isospin or nuclear medium usually interact with the background medium. Therefore, one should regard the non-Abelian fluctuations to represent such isospin interactions. For the meson case, it was shown that the isospin interaction causes the meson mass splitting [20]. One can also easily expect that similar mass splitting occurs in nucleon due to the nonzero isospin charges of nucleons. In this paper, we will investigate the nucleon masses and nucleon-pion coupling in the isospin medium. In this procedure the important and crucial thing is to find appropriate gauge fixing terms which can decouple pion from

the axial vector fluctuations. In the isospin medium, due to the non-Abelian nature of fluctuations, the unitary gauge fixing terms used in the AdS case [7, 32] should be modified. After the careful calculation, we find that the generalized non-Abelian unitary gauge fixing is allowed in the isospin medium which makes us be able to study the nucleon-pion coupling even in the isospin medium. In the non-Abelian unitary gauge, we find that there exists a nucleon mass splitting. The mass shift of nucleon is exactly given by the half of the meson mass splitting because the isospin charge of nucleon is half of the charged meson. Although the isospin interaction shifts meson's and nucleon's masses, it does not affect on their Fourier mode solutions. Using this fact we show that there is no nucleon-pion coupling splitting in the isospin medium.

The rest of the paper is organized as follows: In Sec. 2, we summarize the dual geometry and find the non-Abelian unitary gauge fixing terms in the isospin medium. In this non-Abelian unitary gauge, the meson mass splitting are reinvestigated which gives rise to the consistent results obtained in the axial gauge [20]. In Sec. 3, by using the non-Abelian unitary gauge we investigate the nucleon mass splitting and nucleon-pion coupling in the isospin medium. We conclude our work with some remarks in Sec. 4.

2 Isospin medium in hard wall model

Let us start with reviewing briefly the isospin medium and our conventions. Isospin medium is composed of matter having only the isospin chemical potential without any baryonic net charge. To describe the isospin medium holographically, we should take into account a gravity theory including $SU(2)_L \times SU(2)_R$ flavor gauge group [18, 20]

$$S = \int d^5x \sqrt{G} \text{Tr} \left[\frac{1}{2\kappa^2} (\mathcal{R} - 2\Lambda) - \frac{1}{4g_5^2} F_{MN}^1 F^{1MN} - \frac{1}{4g_5^2} F_{MN}^2 F^{2MN} + |D\Phi|^2 + \frac{3}{R^2} |\Phi|^2 \right], \quad (1)$$

where $\Lambda = -6/R^2$ is a cosmological constant. The gauge field strength are given by

$$\begin{aligned} F_{MN}^1 &= \partial_M L_N - \partial_N L_M - i[L_M, L_N], \\ F_{MN}^2 &= \partial_M R_N - \partial_N R_M - i[R_M, R_N], \end{aligned} \quad (2)$$

and a covariant derivative acting on the complex scalar field is defined as

$$D_M \Phi = \partial_M \Phi - iL_M \Phi + iR_M \Phi. \quad (3)$$

Since the isospin matter can be clarified by the Cartan subalgebra of the flavor group, it is sufficient to turn only on L_t^3 and R_t^3 where 3 and t denote the gauge index and time component respectively. Note that the isospin matter is not an ordinary nucleon matter, because it does not contain any information for the baryon number. Since the isospin chemical potential is given by a constant on the AdS background, there is no gravitational backreaction caused by the isospin matter. So the dual

geometry still remains the thermal AdS space for the confining phase or the Schwarzschild AdS black brane for the deconfining phase. To describe the confinement in the bottom-up approach, one should introduce a hard or soft wall. In this paper, we will concentrate on the hard wall model [5, 20]. In the confining phase, the dual geometry is with an appropriate IR cutoff z_{IR}

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad (4)$$

where $\eta_{\mu\nu} = (+1, -1, -1, -1)$.

The modulus of the complex scalar field ϕ on the AdS background geometry satisfies the following equation of motion

$$0 = -\frac{1}{\sqrt{G}} \partial_z \left(\sqrt{G} G^{zz} \partial_z \phi \right) + 3\phi, \quad (5)$$

where we set $R = 1$. Its solution of ϕ is given by

$$\phi = m_q z + \sigma z^3. \quad (6)$$

In the dual theory point of view, m_q and σ correspond to the quark mass and the chiral condensate respectively. In general, the energy-momentum tensor of the scalar field can modify the background geometry. In this paper, however, we neglect the gravitational backreaction of the scalar field because it corresponds to the $1/N_c$ correction where N_c is the rank of the gauge group [25, 26].

The isospin matter can be described by two diagonal subgroup elements of the flavor group and it can be further decomposed into the symmetric or antisymmetric combinations

$$V_t^3 = \frac{1}{2} (L_t^3 + R_t^3) \quad , \quad A_t^3 = \frac{1}{2} (L_t^3 - R_t^3). \quad (7)$$

Under the parity transformation, $L \leftrightarrow R$, the parity-even state usually has lower energy than the parity-odd one. So we assume that all isospin matters are in the lowest parity-even state satisfying $L_t^3 = R_t^3$, which corresponds to the ground state of the isospin medium. In the nuclear medium [20], the boundary value of V_t^3 corresponds to the isospin chemical potential of quarks

$$V_t^3 = \begin{pmatrix} \mu_u \\ \mu_d \end{pmatrix}. \quad (8)$$

In the isospin medium, μ_u and μ_d imply the isospin chemical potential of quark-like particles without baryon charge. In order to distinguish such particles from ordinary quarks and nucleons, we use a new terminology, iso-particle. Taking the analogy to the nuclear matter, the isospin matter implies a medium composed of iso-nucleons which have three iso-quarks inside similar to ordinary nucleons. Since the fundamental excitations are not quarks but nucleons in the confining phase, we need to reinterpret V_t^3 in terms of iso-nucleon quantities. Although the isospin matter is not the nuclear matter as mentioned previously, one can still use the same definition used in the nuclear matter

because the isospin matter can be regarded as the specific limit of the nuclear matter (see the details in [20]). Then, V_t^3 can be rewritten in terms of the chemical potentials of iso-proton and iso-neutron

$$V_t^3 = \sqrt{2}\pi^2 (\mu_P - \mu_N), \quad (9)$$

where $\mu_P = 2\mu_u + \mu_d$ and $\mu_N = \mu_u + 2\mu_d$.

To investigate the spectra and coupling of nucleons and meson, let us turn on the small fluctuations

$$\begin{aligned} L_M &\equiv L_M^a T^a = (V_M^a + l_M^a) T^a, \\ R_M &\equiv R_M^a T^a = (V_M^a + r_M^a) T^a, \\ \Phi &= \phi e^{iP} = \phi e^{iP^a T^a}, \end{aligned} \quad (10)$$

where l_M^a , r_M^a and P^a are fluctuations and $T^a = \sigma^a/2$ are generators of the $SU(2)$ flavor symmetry group. In the axial gauge $v_z = a_z = 0$, the meson mass splitting in the isospin and nuclear medium has been investigated in [19, 20]. In [7], the meson masses by taking the unitary gauge instead of the axial gauge were investigated which has been further generalized to the nucleon masses and their couplings [32]. In this paper, we will investigate the nucleon masses and coupling in the isospin medium. To do so, it is required to define meson spectra and pion in the isospin medium correctly in the unitary gauge. Note that in the axial gauge $v_z = a_z = 0$ [5, 20], P was identified with the pion field. On the other hand, authors of [7, 32] have introduced gauge-fixing terms without taking $a_z = 0$ and chosen the unitary gauge ($\xi_{v,a} \rightarrow \infty$). In the latter case, the nonzero a_z was identified with pion. Here, we will follow the same strategy used in the unitary gauge. In the isospin medium, due to the background gauge field V_t^3 and non-Abelian fluctuations, the unitary gauge fixing term should be modified. To find the correct gauge fixing term in the isospin medium, we first expand the action to quadratic order. After some calculations together with the following redefinition

$$v_M = \frac{1}{\sqrt{2}}(l_M + r_M) \quad \text{and} \quad a_M = \frac{1}{\sqrt{2}}(l_M - r_M), \quad (11)$$

the action at quadratic order can be rewritten as

$$S = S_v + S_a, \quad (12)$$

with

$$\begin{aligned} S_v &= -\frac{1}{2g_5^2} \int d^5x \sqrt{G} \left[G^{\mu\nu} G^{\rho\sigma} F_{\mu\rho}^{(v)i} F_{\nu\sigma}^{(v)i} + 2 (\mathcal{D}_\mu v_z^i)^2 - 4G^{\mu\nu} G^{zz} (\partial_z v_\mu^i) (\mathcal{D}_\nu v_z^i) \right. \\ &\quad \left. + 2 (\partial_z v_\mu^i)^2 \right], \end{aligned} \quad (13)$$

$$\begin{aligned} S_a &= \int d^5x \sqrt{G} \left[-\frac{1}{2g_5^2} \left\{ G^{\mu\nu} G^{\rho\sigma} F_{\mu\rho}^{(a)i} F_{\nu\sigma}^{(a)i} + 2 (\mathcal{D}_\mu a_z^i)^2 - 4G^{\mu\nu} G^{zz} (\partial_z a_\mu^i) (\mathcal{D}_\nu a_z^i) + 2 (\partial_z a_\mu^i)^2 \right\} \right. \\ &\quad \left. + \frac{\phi^2}{2} (\mathcal{D}_\mu P^i - \sqrt{2}a_\mu^i)^2 + \frac{\phi^2}{2} (\partial_z P^i - \sqrt{2}a_z^i)^2 \right], \end{aligned} \quad (14)$$

where $i = 1, 2, 3$ are the flavor group indices. Since we consider the non-Abelian fluctuations, the covariant derivative \mathcal{D}_μ instead of a normal derivative generally acts on fluctuations, for example,

$$\begin{aligned}\mathcal{D}_M P^i &= \partial_M P^i + \epsilon^{ij3} V_M^3 P^j, \\ F_{MN}^{(v)i} &= \mathcal{D}_M v_N^i - \mathcal{D}_N v_M^i, \\ F_{MN}^{(a)i} &= \mathcal{D}_M a_N^i - \mathcal{D}_N a_M^i,\end{aligned}\tag{15}$$

where ϵ^{ijk} is the structure constant of the $SU(2)$ flavor group. Note that $\mathcal{D}_z = \partial_z$ because of $V_z^i = 0$. The fluctuation action includes several mixing terms which can be removed by choosing an appropriate gauge fixing. In [7, 32], the Abelian-type gauge fixing terms in the gluon medium without matter have been found. In the isospin medium, due to the isospin interaction, we should turned on the non-Abelian fluctuations for which we need to generalize the Abelian gauge fixing to the non-Abelian one. We find that the following non-Abelian gauge fixing terms can get rid of most of mixing terms

$$\begin{aligned}S_{GF} &= \int d^5x \left[-\frac{1}{2g_5^2 z \xi_v} \left\{ \mathcal{D}^\mu v_\mu^i - \xi_v z \partial_z \left(\frac{v_z^i}{z} \right) \right\}^2 \right. \\ &\quad \left. - \frac{1}{2g_5^2 z \xi_a} \left\{ \mathcal{D}^\mu a_\mu^i - \xi_a z \partial_z \left(\frac{a_z^i}{z} \right) + \xi_a \frac{2\sqrt{2}g_5^2 \phi^2}{z^2} P^i \right\}^2 \right].\end{aligned}\tag{16}$$

Note that this non-Abelian generalization is possible because of the constant gauge potential V_t^3 . In the unitary gauge ($\xi_{v,a} \rightarrow \infty$), v_z^i is decoupled from the theory. Furthermore, if the following relation is satisfied

$$P^i = \frac{z^3}{2\sqrt{2} g_5^2 \phi^2} \partial_z \left(\frac{a_z^i}{z} \right),\tag{17}$$

the orthogonal combination of a_z and P remains massless similar to [7, 32].

Now, let us take into account the meson spectra in the non-Abelian unitary gauge in which the action for the vector and axial-vector fluctuations simply reduce to

$$\begin{aligned}S_v &= -\frac{1}{2g_5^2} \int d^5x \sqrt{G} \left(G^{\mu\nu} G^{\rho\sigma} F_{\mu\rho}^{v(i)} F_{\nu\sigma}^{v(i)} + 2G^{zz} G^{\mu\nu} \partial_z v_\mu^i \partial_z v_\nu^i \right), \\ S_a &= -\frac{1}{2g_5^2} \int d^5x \sqrt{G} \left(G^{\mu\nu} G^{\rho\sigma} F_{\mu\rho}^{a(i)} F_{\nu\sigma}^{a(i)} + 2G^{zz} G^{\mu\nu} \partial_z a_\mu^i \partial_z a_\nu^i - 4g_5^2 \phi^2 G^{\mu\nu} a_\mu^i a_\nu^i \right),\end{aligned}\tag{18}$$

The nonzero value of the background gauge field in the isospin medium breaks the boost symmetry so that the time component fluctuations usually behaves differently from the spatial components. Moreover, since the time component fluctuations are associated with the isospin charge rather than the meson spectra, we turn off the time component fluctuations, $v_0^i = a_0^i = 0$, from now on and focus only on the spatial components. In order to identify the bulk fluctuations with mesons of the dual field theory, we introduce the ρ - and a_1 -meson notations

$$\begin{aligned}v_\mu^1(z, \vec{x}) &= \frac{1}{\sqrt{2}} (\rho_\mu^+ + \rho_\mu^-), & v_\mu^2(z, \vec{x}) &= \frac{i}{\sqrt{2}} (\rho_\mu^+ - \rho_\mu^-), & v_\mu^3(z, \vec{x}) &= \rho_\mu^0, \\ a_\mu^1(z, \vec{x}) &= \frac{1}{\sqrt{2}} (a_{1\mu}^+ + a_{1\mu}^-), & a_\mu^2(z, \vec{x}) &= \frac{i}{\sqrt{2}} (a_{1\mu}^+ - a_{1\mu}^-), & a_\mu^3(z, \vec{x}) &= a_{1\mu}^0,\end{aligned}\tag{19}$$

where the superscripts, \pm and 0, imply the isospin charges of mesons. Using the Fourier mode expansions in the rest frame

$$\begin{aligned}\rho_\nu^0(z, \vec{x}) &= \int \frac{d\omega_0}{2\pi} e^{i\omega_0 t} \rho_\nu^0(z, \omega_0), \\ \rho_\nu^\pm(z, \vec{x}) &= \int \frac{d\omega_0}{2\pi} e^{i\omega_\pm t} \rho_\nu^\pm(z, \omega_\pm),\end{aligned}\tag{20}$$

ρ -mesons satisfy

$$\begin{aligned}0 &= \partial_z \left(\sqrt{G} G^{zz} G^{\mu\nu} \partial_z \rho_\nu^0 \right) - \omega_0^2 \sqrt{G} G^{00} G^{\mu\nu} \rho_\nu^0, \\ 0 &= \partial_z \left(\sqrt{G} G^{zz} G^{\mu\nu} \partial_z \rho_\nu^\pm \right) - (\omega_\pm \mp V_t^3)^2 \sqrt{G} G^{00} G^{\mu\nu} \rho_\nu^\pm,\end{aligned}\tag{21}$$

where ρ_ν^0 and ρ_ν^\pm are Fourier modes. To solve these second order differential equations, one should impose two boundary conditions: one is the Dirichlet boundary condition at the asymptotic boundary, $\rho_\nu^0|_{z=0} = \rho_\nu^\pm|_{z=0} = 0$, and the other is the Neumann boundary condition at the IR cutoff, $\partial_z \rho_\nu^0|_{z=z_{IR}} = \partial_z \rho_\nu^\pm|_{z=z_{IR}} = 0$. Since ρ_ν^0 and ρ_ν^\pm satisfy the same boundary conditions and V_t^3 is a constant, the comparison of two differential equations shows that the charged ρ -meson masses are related to the neutral one

$$\omega_\pm = \omega_0 \pm \sqrt{2}\pi^2 (\mu_P - \mu_N).\tag{22}$$

This result is consistent with those in [20] where the axial gauge was used. Inserting the mass relation into the above differential equations, one can easily see that ρ_ν^\pm should be the same as ρ_ν^0 . Similarly, the Fourier mode expansions of a_1 -meson

$$\begin{aligned}a_{1\nu}^0(z, \vec{x}) &= \int \frac{d\bar{\omega}_0}{2\pi} e^{i\bar{\omega}_0 t} a_{1\nu}^0(z, \bar{\omega}_0), \\ a_{1\nu}^\pm(z, \vec{x}) &= \int \frac{d\bar{\omega}_0}{2\pi} e^{i\bar{\omega}_\pm t} a_{1\nu}^\pm(z, \bar{\omega}_\pm),\end{aligned}\tag{23}$$

lead to

$$\begin{aligned}0 &= \partial_z \left(\sqrt{G} G^{zz} G^{\mu\nu} \partial_z a_{1\nu}^0 \right) - (G^{00} \bar{\omega}_0^2 - 2g_5^2 \phi^2) \sqrt{G} G^{\mu\nu} a_{1\nu}^0, \\ 0 &= \partial_z \left(\sqrt{G} G^{zz} G^{\mu\nu} \partial_z a_{1\nu}^\pm \right) - \left(G^{00} (\bar{\omega}_\pm \mp V_t^3)^2 - 2g_5^2 \phi^2 \right) \sqrt{G} G^{\mu\nu} a_{1\nu}^\pm.\end{aligned}\tag{24}$$

The same boundary conditions used in the ρ -meson give rise to the mass relation of a_1 -mesons

$$\bar{\omega}_\pm = \bar{\omega}_0 \pm \sqrt{2}\pi^2 (\mu_P - \mu_N).\tag{25}$$

Like ρ -meson, the same boundary conditions yields $a_{1\nu}^\pm = a_{1\nu}^0$.

The action for the pseudoscalar field in the unitary gauge is given by

$$S_s = \int d^5x \sqrt{G} \left[-\frac{1}{2g_5^2} G^{zz} \mathcal{D}_\mu a_z^i \mathcal{D}^\mu a_z^i + \frac{\phi^2}{2} \mathcal{D}_\mu P^i \mathcal{D}^\mu P^i + \frac{\phi^2}{2} \left(\partial_z P^i - \sqrt{2} a_z^i \right)^2 \right].\tag{26}$$

If the pion field is identified with a component of a_z

$$a_z^i(x, z) = f_0^i(z) \pi^i(x). \quad (27)$$

the last terms in (26) can be removed by requiring an additional relation [32]

$$0 = \partial_z \left[\frac{z^3}{\phi^2} \partial_z \left(\frac{f_0^i}{z} \right) \right] - 4g_5^2 f_0^i. \quad (28)$$

Since the mode functions of pion satisfy the same equation, the same boundary conditions lead to the same solutions. If we set

$$f_0 \equiv f_0^1 = f_0^2 = f_0^3, \quad (29)$$

the following analytic solution in the chiral limit ($m_q = 0$) is allowed with two integration constants, N and c ,

$$f_0 = N z^3 \left[I_{2/3} (2\sigma g_5 z^3/3) - c I_{-2/3} (2\sigma g_5 z^3/3) \right], \quad (30)$$

where $I_{\pm 2/3}$ are the modified Bessel functions. In [37], c has been fixed by imposing the Dirichlet boundary condition, $f_0(z_{IR}) = 0$,

$$c = \frac{I_{2/3} (2\sigma g_5 z_{IR}^3/3)}{I_{-2/3} (2\sigma g_5 z_{IR}^3/3)}, \quad (31)$$

and the overall normalization constant N was fixed by the normalization condition [32, 36]

$$1 = \int_0^{z_{IR}} dz \left[\frac{1}{2g_5^2 z} f_0^2 + \frac{z^3}{16\phi^2 g_5^4} \left\{ \partial_z \left(\frac{f_0}{z} \right) \right\}^2 \right]. \quad (32)$$

Using $1/\kappa^2 = N_c^2/(4\pi^2 R^3)$ and $1/g_5^2 = N_c^2/(4\pi^2 R)$ together with $N_c = 3$, the numerical value of the normalization constant reads $N = 0.3781$.

3 Nucleons in the isospin medium

As shown in the previous section, the background isospin matter shifts the masses of the charged mesons. So one can easily expect that there is also similar mass shift in the nucleon mass because nucleons have also nonzero isospin charges. In this section, we will investigate the nucleon mass splitting in the isospin medium. To do so, we take into account the fermionic fluctuations on the 5-dimensional thermal AdS background

$$S = \int d^5x \sqrt{G} \left[i\bar{\Psi}^1 \Gamma^M \nabla_M \Psi^1 + i\bar{\Psi}^2 \Gamma^M \nabla_M \Psi^2 - m_1 \bar{\Psi}^1 \Psi^1 - m_2 \bar{\Psi}^2 \Psi^2 - g_Y \left(\bar{\Psi}^1 \Phi \Psi^2 + \bar{\Psi}^2 \Phi^+ \Psi^1 \right) \right], \quad (33)$$

where ∇_M denotes a covariant derivative including the spin and gauge connections

$$\nabla_M \Psi^{(1,2)} = \left(\partial_M - \frac{i}{4} \omega_M^{AB} \Gamma_{AB} - iV_M \right) \Psi^{(1,2)}, \quad (34)$$

where $\Gamma^{AB} = \frac{1}{2i} [\Gamma^A, \Gamma^B]$ and $V_t^3 = L_t^3 = R_t^3$ has been used. Here, $(1, 2)$ implies either 1 or 2 and g_Y is the Yukawa coupling. In the above action, Ψ^1 and Ψ^2 transform as $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ under the flavor group $SU(2)_L \times SU(2)_R$. Due to the Yukawa term, Ψ^1 and Ψ^2 are coupled to each other and then the chiral symmetry is broken down. This chirality can be also related to the chirality of the boundary fermion, proton and neutron. To do that, one should take $m_1 = 5/2$ and at the same time $m_2 = -5/2$. Then, following the AdS/CFT relation

$$m_{(1,2)}^2 = (\Delta - 2)^2, \quad (35)$$

all dual fermionic operators of Ψ^1 and Ψ^2 have the conformal dimension of nucleon, $\Delta = 9/2$.

If ignoring the Yukawa term, the variation of action leads to the Dirac equation

$$[i\Gamma^M \nabla_M - m_{(1,2)}] \Psi^{(1,2)} = 0 \quad (36)$$

and the following boundary term

$$\delta \bar{\Psi}^{(1,2)} \Gamma^M \Psi^{(1,2)} \Big|_{\epsilon}^{z_{IR}} = 0, \quad (37)$$

where z_{IR} and ϵ are the IR and UV cutoff respectively. These are equations on the curved manifold, so it is more convenient to introduce quantities defined on the tangent manifold in order to solve the Dirac equation. The vielbein e_M^A of the thermal AdS space is chosen as $e_M^A = \frac{1}{z} \delta_M^A$, where A, B and M, N are indices of the tangent and curved manifold respectively. The non-zero components of spin connection ω_M^{AB} are given by $\omega_M^{5A} = \frac{1}{z} \delta_M^A$. In addition, we choose the following gamma matrix on the tangent space

$$\Gamma^t = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \Gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \Gamma^z = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad (38)$$

where σ^i is the Pauli matrix. If we further define the 4-dimensional gamma matrix $\gamma^\mu = \Gamma^\mu$ ($\mu = 0, 1, 2, 3$), then the 4-dimensional chiral operator is given by $\gamma^5 = i\Gamma^z$. Using the vielbein, the above Dirac equation simply reduces to

$$\begin{aligned} 0 &= \left[i e_A^M \Gamma^A \left(\partial_M - \frac{i}{4} \omega_M^{AB} \Gamma_{AB} - i V_M \right) - m_1 \right] \Psi^1 - g_Y \phi \Psi^2, \\ 0 &= \left[i e_A^M \Gamma^A \left(\partial_M - \frac{i}{4} \omega_M^{AB} \Gamma_{AB} - i V_M \right) - m_2 \right] \Psi^2 - g_Y \phi \Psi^1, \end{aligned} \quad (39)$$

where ϕ is the modulus of Φ in (6).

Introducing the Fourier mode of fermion

$$f^{(1,2)}(p, z) \psi^{(1,2)}(p) = \int d^4x \Psi^{(1,2)}(x, z) e^{-ipx}, \quad (40)$$

and the Weyl spinor representation with $\psi_L^{(1,2)} = \gamma^5 \psi_L^{(1,2)}$ and $\psi_R^{(1,2)} = -\gamma^5 \psi_R^{(1,2)}$

$$\psi^{(1,2)} = \begin{pmatrix} \psi_L^{(1,2)} \\ \psi_R^{(1,2)} \end{pmatrix}, \quad (41)$$

the 4-dimensional Weyl spinors satisfy

$$\not{p} \psi_{L,R}^{(1,2)}(p) = |p| \psi_{R,L}^{(1,2)}(p), \quad (42)$$

where the subscripts, L or R , denote the 4-dimensional chirality. If one takes the normalizable modes as f_L^1 and f_R^2 for Ψ_L^1 and Ψ_R^2 respectively, the chirality of the $SU(2)_L \times SU(2)_R$ flavor group can be associated with the 4-dimensional chirality. In terms of the Weyl fermions, the equations in (39) are reduced to

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{g_Y \phi}{z} \\ -\frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_L^1 \\ f_L^2 \end{pmatrix} = -(|p| - V_t) \begin{pmatrix} f_R^1 \\ f_R^2 \end{pmatrix}, \quad (43)$$

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & \frac{g_Y \phi}{z} \\ \frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_R^2 \\ f_R^1 \end{pmatrix} = (|p| - V_t) \begin{pmatrix} f_L^2 \\ f_L^1 \end{pmatrix}. \quad (44)$$

These matrix equations can be further reduces to the symmetric and anti-symmetry combinations which describe the parity-even and parity-odd excitations under $1 \leftrightarrow 2$ and simultaneously $L \leftrightarrow R$ transformation. As a result, the lowest nucleon spectra corresponding to proton and neutron, which are parity-even states, are described by the lowest excitation of the symmetric combination $f_L^1 + f_R^2$ together with $f_R^1 - f_L^2$. On the other hand, the parity-odd states are represented as $f_L^1 - f_R^2$ and $f_R^1 + f_L^2$. In order to investigate the parity-even mass spectra, one can impose $f_L^1 = f_R^2$ and $f_R^1 = -f_L^2$, then the above two matrix equations, (43) and (44), reduce to the same matrix equation

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & \frac{g_Y \phi}{z} \\ \frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_L^1 \\ f_R^1 \end{pmatrix} = \begin{pmatrix} -(|p| - V_t) & 0 \\ 0 & |p| - V_t \end{pmatrix} \begin{pmatrix} f_R^1 \\ f_L^1 \end{pmatrix}. \quad (45)$$

Similarly, imposing $f_L^1 = -f_R^2$ and $f_R^1 = f_L^2$ gives rise to the matrix equation for the parity-odd states

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{g_Y \phi}{z} \\ -\frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_L^1 \\ f_R^1 \end{pmatrix} = \begin{pmatrix} -(|p| - V_t) & 0 \\ 0 & |p| - V_t \end{pmatrix} \begin{pmatrix} f_R^1 \\ f_L^1 \end{pmatrix}. \quad (46)$$

In this case, the parity-even and parity-odd states are distinguished due to the Yukawa coupling caused by the modulus of the complex scalar field. Furthermore, since the nucleon has an isospin charge, it can interact with the background isospin matter. If the n -th excitation mode of $f_{L,R}^1$ is denoted by $f_{L,R}^{1(n,\pm,\pm)}$ where the first and second sign imply the parity and isospin quantum number respectively, the parity-even state satisfying (45) can be further decomposed, depending on the isospin charge, into

$$\begin{aligned} \begin{pmatrix} \partial_z - \frac{\Delta}{z} & \frac{g_Y \phi}{z} \\ \frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_L^{1(n,+,+)} \\ f_R^{1(n,+,+)} \end{pmatrix} &= \begin{pmatrix} -\left(|p| - \frac{V_t^3}{2}\right) & 0 \\ 0 & |p| - \frac{V_t^3}{2} \end{pmatrix} \begin{pmatrix} f_R^{1(n,+,+)} \\ f_L^{1(n,+,+)} \end{pmatrix}, \\ \begin{pmatrix} \partial_z - \frac{\Delta}{z} & \frac{g_Y \phi}{z} \\ \frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_L^{1(n,+,-)} \\ f_R^{1(n,+,-)} \end{pmatrix} &= \begin{pmatrix} -\left(|p| + \frac{V_t^3}{2}\right) & 0 \\ 0 & |p| + \frac{V_t^3}{2} \end{pmatrix} \begin{pmatrix} f_R^{1(n,+,-)} \\ f_L^{1(n,+,-)} \end{pmatrix}. \end{aligned} \quad (47)$$

On the other hand, the parity-odd states are governed by

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{g_Y \phi}{z} \\ -\frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_L^{1(n,-,+)} \\ f_R^{1(n,-,+)} \end{pmatrix} = \begin{pmatrix} -\left(|p| - \frac{V_t^3}{2}\right) & 0 \\ 0 & |p| - \frac{V_t^3}{2} \end{pmatrix} \begin{pmatrix} f_R^{1(n,-,+)} \\ f_L^{1(n,-,+)} \end{pmatrix},$$

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{g_Y \phi}{z} \\ -\frac{g_Y \phi}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_L^{1(n,-,-)} \\ f_R^{1(n,-,-)} \end{pmatrix} = \begin{pmatrix} -\left(|p| + \frac{V_t^3}{2}\right) & 0 \\ 0 & |p| + \frac{V_t^3}{2} \end{pmatrix} \begin{pmatrix} f_R^{1(n,-,-)} \\ f_L^{1(n,-,-)} \end{pmatrix}. \quad (48)$$

The lowest excitation modes, proton and neutron, can be represented by $f_{L,R}^{(1,+,+)}$ and $f_{L,R}^{(1,+,-)}$. Since the lowest excitations have only the parity-even states, there is no $f_{L,R}^{(1,-,\pm)}$. For the higher resonances, they can have even and odd parity states.

From the above matrix equations, one can easily derive two second order differential equations for $f_L^{1(n,\pm,\pm)}$

$$0 = \left[\partial_z^2 - \frac{4}{z} \partial_z + \left\{ \frac{(5-\Delta)\Delta}{z^2} - \frac{g_Y^2 \phi^2}{z^2} + \left(|p| - \frac{V_t^3}{2} \right)^2 \right\} \right] f_L^{1(n,\pm,+)}, \quad (49)$$

$$0 = \left[\partial_z^2 - \frac{4}{z} \partial_z + \left\{ \frac{(5-\Delta)\Delta}{z^2} - \frac{g_Y^2 \phi^2}{z^2} + \left(|p| + \frac{V_t^3}{2} \right)^2 \right\} \right] f_L^{1(n,\pm,-)}. \quad (50)$$

The mass of nucleon is given by p satisfying the following two boundary conditions

$$f_L^{1(n,\pm,\pm)}(0) = 0 \quad \text{and} \quad f_R^{1(n,\pm,\pm)}(z_{IR}) = 0, \quad (51)$$

in which the boundary terms in (37) vanish. In [32], the best parameters fitting the lowest nucleon masses were given by $z_{IR} = 1/(0.205 \text{ GeV})$ and $g_Y = 14.4$ and several nucleon masses have been investigated for $V_t^3 = 0$. In the meson spectra of the hard wall model, the higher excitation modes usually have quite different masses from the phenomenological data. In the nucleon mass the same thing happens [32].

Now, let us take into account the mass splitting of nucleons in the isospin medium. Assuming that the nucleon mass is given by p_0 for $V_t^3 = 0$ in which proton and neutron are indistinguishable. In the isospin medium with a nonzero V_t^3 , the proton and neutron masses are shifted due to the isospin interaction. Since the isospin medium has a constant V_t^3 , the boundary conditions in (51) lead to $p_0 = p_P - V_t^3/2$ for proton and $p_0 = p_N + V_t^3/2$ for neutron. As a result, the proton and neutron masses in the isospin medium are given by

$$\begin{aligned} p_P &= p_0 + \frac{\pi^2}{\sqrt{2}} (\mu_P - \mu_N), \\ p_N &= p_0 - \frac{\pi^2}{\sqrt{2}} (\mu_P - \mu_N), \end{aligned} \quad (52)$$

where p_0 generally depends on z_{IR} , g_Y , m_q and σ . This result shows that the proton (or neutron) mass increases (or decreases) when the isospin chemical potential difference, $\mu_P - \mu_N$, grows. As a result,

the isospin interaction between nucleon and the isospin matter shifts the nucleon mass so that like the charged meson case there exists a nucleon mass splitting proportional to the total isospin chemical potential of the isospin matter. Since nucleon has the half isospin charge of the charged meson the nucleon mass shift, when $\mu_P - \mu_N$ is given, is exactly half of the meson mass shift. Related solutions of (47) and (48), there is a remarkable point. After inserting the mass relations in (52) back into the equations of motion, (47) and (48), one can easily find

$$f_L^{1(m,\pm,+)} = f_L^{1(m,\pm,-)} \quad \text{and} \quad f_R^{1(m,\pm,+)} = f_R^{1(m,\pm,-)}, \quad (53)$$

which implies that in spite of the nucleon mass shiftings the Fourier mode functions of nucleons are not changed. In the chiral limit with $m_q = 0$ and $\sigma \neq 0$, the nucleon mass shifts are checked numerically in Fig. 1. Here we use $z_{IR} = 1/(0.3227 \text{ GeV})$, $m_q = 0$ and $\sigma = (0.304 \text{ GeV})^3$, which gave rise to good lowest meson spectra in [20], and take $g_Y = 4.762$ to obtain a correct mass of proton and neutron, $p_0 = 0.9399 \text{ GeV}$, for $\mu_P - \mu_N = 0$.

Another interesting limit is the chiral phase transition point where the chiral condensate disappears and ϕ simply reduces to $\phi = m_q z$. If the nucleon mass is given by m_0 for $m_q = V_t^3 = 0$, the proton mass from (49) yields

$$p_P = \sqrt{m_0^2 + g_Y^2 m_q^2} + \frac{\pi^2}{\sqrt{2}} (\mu_P - \mu_N), \quad (54)$$

whereas the neutron mass from (50) is given by

$$p_N = \sqrt{m_0^2 + g_Y^2 m_q^2} - \frac{\pi^2}{\sqrt{2}} (\mu_P - \mu_N). \quad (55)$$

These results, as expected, show that proton becomes more massive in the isospin medium with $\mu_P \gg \mu_N$ so that in this case it is easier to create neutron than proton.

From the fermion action (33), the interaction term between nucleons and pion at cubic order is described by

$$S_{int} = i \int d^4x \int_0^{z_{IR}} \frac{dz}{z^4} \left[\left(\psi_R^{1+} l_z \psi_L^1 - \psi_L^{1+} l_z \psi_R^1 + \psi_R^{2+} r_z \psi_L^2 - \psi_L^{2+} r_z \psi_R^2 \right) + g_Y \phi \left(\psi_R^{1+} P \psi_L^2 + \psi_L^{1+} P \psi_R^2 - \psi_R^{2+} P \psi_L^1 - \psi_L^{2+} P \psi_R^1 \right) \right], \quad (56)$$

where l_z and r_z are left and right gauge fluctuations of $SU(2)_L \times SU(2)_R$ in the z -direction. Using (11), v_z is decoupled in the unitary gauge as mentioned before. Introducing further the following notations

$$\begin{pmatrix} \psi_L^1 \\ \psi_L^2 \end{pmatrix} = \begin{pmatrix} f_L^{1(n,\pm,\pm)} \psi_L^{(n,\pm,\pm)} \\ f_L^{2(n,\pm,\pm)} \psi_L^{(n,\pm,\pm)} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \psi_R^1 \\ \psi_R^2 \end{pmatrix} = \begin{pmatrix} f_R^{1(n,\pm,\pm)} \psi_R^{(n,\pm,\pm)} \\ f_R^{2(n,\pm,\pm)} \psi_R^{(n,\pm,\pm)} \end{pmatrix}, \quad (57)$$

the parity-even and parity-odd modes satisfy

$$\begin{aligned} f_L^{1(n,+, \pm)} &= f_R^{2(n,+, \pm)}, & f_R^{1(n,+, \pm)} &= -f_L^{2(n,+, \pm)}, \\ f_L^{1(n,-, \pm)} &= -f_R^{2(n,-, \pm)}, & f_R^{1(n,-, \pm)} &= f_L^{2(n,-, \pm)}. \end{aligned} \quad (58)$$

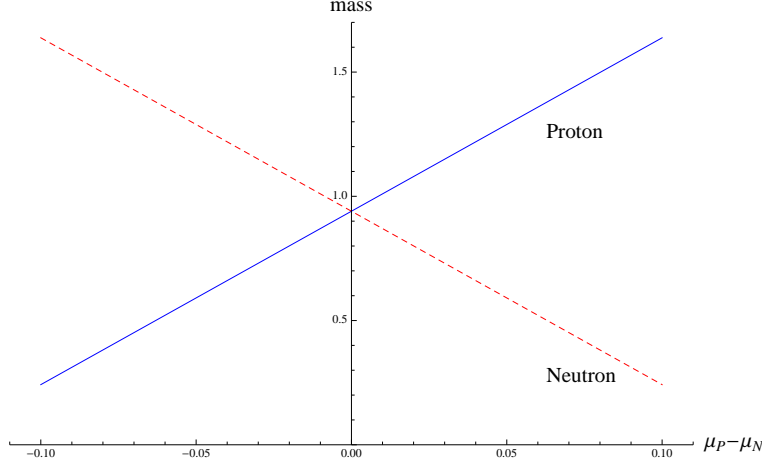


Figure 1: The masses of proton and neutron in the isospin medium where we use $z_{IR} = 1/(0.3227 \text{ GeV})$, $m_q = 0$ and $\sigma = (0.304 \text{ GeV})^3$. For $\mu_P - \mu_N = 0$, proton and neutron are degenerate and their mass, 0.94 GeV, is well evaluated with $g_Y = 4.762$.

Assuming that two nucleons have the same parity, the interaction term between two nucleons and pion leads to

$$S_{int} = i\sqrt{2} \int d^4x g_{(m,\pm,\pm;n,\pm,\pm)} \bar{\Psi}^{(m,\pm,\pm)} \gamma^5 \pi \Psi^{(n,\pm,\pm)}, \quad (59)$$

where $g_{(m,\pm,\pm;n,\pm,\pm)}$ denote the nucleons-pion coupling and $\Psi^{(m,\pm,\pm)}$ implies the 4-dimensional Dirac fermion

$$\Psi^{(n,\pm,\pm)} = \begin{pmatrix} \psi_L^{(n,\pm,\pm)} \\ \psi_R^{(n,\pm,\pm)} \end{pmatrix}, \quad (60)$$

and $\pi = \pi^a t^a$ with the diagonal $SU(2)$ flavor group generator t^a . On the other hand, if two nucleons have opposite parity, the interaction term is given by

$$S_{int} = i\sqrt{2} \int d^4x g_{(m,\pm,\pm;n,\mp,\pm)} \bar{\Psi}^{(m,\pm,\pm)} \pi \Psi^{(n,\mp,\pm)}. \quad (61)$$

Since pion and γ^5 have an odd parity, the resulting nucleons-pion interactions seem to be natural. In these cases, the corresponding nucleons-pion coupling are given by

$$g_{(m,+, \pm; n, +, \pm)} = \int_0^{z_{IR}} \frac{dz}{z^4} \left[-\frac{f_0}{2} \left(f_L^{1(m,+, \pm)*} f_R^{1(n,+, \pm)} + f_R^{1(m,+, \pm)*} f_L^{1(n,+, \pm)} \right) + \frac{g_Y z^3}{4g_5^2 \phi} \partial_z \left(\frac{f_0}{z} \right) \left(f_L^{1(m,+, \pm)*} f_L^{1(n,+, \pm)} + f_R^{1(m,+, \pm)*} f_R^{1(n,+, \pm)} \right) \right], \quad (62)$$

$$g_{(m,-, \pm; n, -, \pm)} = \int_0^{z_{IR}} \frac{dz}{z^4} \left[-\frac{f_0}{2} \left(f_L^{1(m,-, \pm)*} f_R^{1(n,-, \pm)} + f_R^{1(m,-, \pm)*} f_L^{1(n,-, \pm)} \right) - \frac{g_Y z^3}{4g_5^2 \phi} \partial_z \left(\frac{f_0}{z} \right) \left(f_L^{1(m,-, \pm)*} f_L^{1(n,-, \pm)} + f_R^{1(m,-, \pm)*} f_R^{1(n,-, \pm)} \right) \right], \quad (63)$$

$$\begin{aligned}
g_{(m,+, \pm; n, -, \pm)} &= \int_0^{z_{IR}} \frac{dz}{z^4} \left[\frac{f_0}{2} \left(f_L^{1(m,+, \pm)*} f_R^{1(n, -, \pm)} - f_R^{1(m,+, \pm)*} f_L^{1(n, -, \pm)} \right) \right. \\
&\quad \left. + \frac{g_Y z^3}{4g_5^2 \phi} \partial_z \left(\frac{f_0}{z} \right) \left(f_L^{1(m,+, \pm)*} f_L^{1(n, -, \pm)} - f_R^{1(m,+, \pm)*} f_R^{1(n, -, \pm)} \right) \right], \quad (64)
\end{aligned}$$

$$\begin{aligned}
g_{(m, -, \pm; n, +, \pm)} &= \int_0^{z_{IR}} \frac{dz}{z^4} \left[\frac{f_0}{2} \left(f_L^{1(m, -, \pm)*} f_R^{1(n, +, \pm)} - f_R^{1(m, -, \pm)*} f_L^{1(n, +, \pm)} \right) \right. \\
&\quad \left. - \frac{g_Y z^3}{4g_5^2 \phi} \partial_z \left(\frac{f_0}{z} \right) \left(f_L^{1(m, -, \pm)*} f_L^{1(n, +, \pm)} - f_R^{1(m, -, \pm)*} f_R^{1(n, +, \pm)} \right) \right], \quad (65)
\end{aligned}$$

where the parity relations in (58) are used and f_0 means the Fourier mode function of pions in (30). Note that the interaction terms can be further decomposed according to the isospin charges of nucleons. For example, if one defines

$$\pi^1 = \frac{1}{\sqrt{2}} (\pi^+ + \pi^-), \quad \pi^2 = \frac{i}{\sqrt{2}} (\pi^+ - \pi^-) \quad \text{and} \quad \pi^3 = \sqrt{2} \pi^0, \quad (66)$$

the isospin charge conservation allows only specific combinations for $\bar{\Psi} \gamma^5 \pi \Psi$

$$\begin{aligned}
\bar{\Psi}^{(m, \pm, +)} \gamma^5 \pi \Psi^{(n, \pm, +)} &\rightarrow \frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, +)} \gamma^5 \pi^0 \Psi^{(n, \pm, +)}, \\
\bar{\Psi}^{(m, \pm, -)} \gamma^5 \pi \Psi^{(n, \pm, -)} &\rightarrow -\frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, -)} \gamma^5 \pi^0 \Psi^{(n, \pm, -)}, \\
\bar{\Psi}^{(m, \pm, +)} \gamma^5 \pi \Psi^{(n, \pm, -)} &\rightarrow \frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, +)} \gamma^5 \pi^+ \Psi^{(n, \pm, -)}, \\
\bar{\Psi}^{(m, \pm, -)} \gamma^5 \pi \Psi^{(n, \pm, +)} &\rightarrow \frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, -)} \gamma^5 \pi^- \Psi^{(n, \pm, +)}, \quad (67)
\end{aligned}$$

and for $\bar{\Psi} \pi \Psi$

$$\begin{aligned}
\bar{\Psi}^{(m, \pm, +)} \pi \Psi^{(n, \mp, +)} &\rightarrow \frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, +)} \pi^0 \Psi^{(n, \mp, +)}, \\
\bar{\Psi}^{(m, \pm, -)} \pi \Psi^{(n, \mp, -)} &\rightarrow -\frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, -)} \pi^0 \Psi^{(n, \mp, -)}, \\
\bar{\Psi}^{(m, \pm, +)} \pi \Psi^{(n, \mp, -)} &\rightarrow \frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, +)} \pi^+ \Psi^{(n, \mp, -)}, \\
\bar{\Psi}^{(m, \pm, -)} \pi \Psi^{(n, \mp, +)} &\rightarrow \frac{1}{\sqrt{2}} \bar{\Psi}^{(m, \pm, -)} \pi^- \Psi^{(n, \mp, +)}. \quad (68)
\end{aligned}$$

First, let us take into account the coupling between the lowest nucleons, proton and neutron, and pion. Since the lowest nucleons are parity-even, only $g_{(1,+, \pm; 1,+, \pm)}$ are allowed. As mentioned in (53), the same mode function of nucleons in the isospin medium, $f_L^{1(m, \pm, \pm)}$ and $f_R^{1(m, \pm, \pm)}$, makes the nucleon-pion coupling independent of the isospin charges and the total isospin chemical potential of the isospin medium. Therefore, in the isospin medium there is no nucleon-pion coupling splitting while the nucleon and meson masses are splitted due to the isospin interaction. This fact is also true

for the higher nucleon resonances. Denoting proton and neutron as $P = \Psi^{(1,+,+)}$ and $N = \Psi^{(1,+,-)}$ respectively and defining

$$g_{\pi NN} \equiv g_{(1,+,+;1,+,+)} = g_{(1,+,+;1,+,-)} = g_{(1,+,+;1,+,-)} = g_{(1,+,+;1,+,+)} , \quad (69)$$

the allowed interaction terms are in the isospin medium

$$S_{int} = i \int d^4x \left[g_{\pi NN} (\bar{P} \gamma^5 \pi^0 P - \bar{N} \gamma^5 \pi^0 N + \bar{P} \gamma^5 \pi^+ N + \bar{N} \gamma^5 \pi^- P) + \dots \right] , \quad (70)$$

where the ellipsis implies interactions with higher nucleon resonances. Using the following normalization condition

$$1 = \int_0^{z_{IR}} \frac{dz}{z^4} \left(\left| f_L^{1(m,\pm,\pm)} \right|^2 + \left| f_R^{1(m,\pm,\pm)} \right|^2 \right) , \quad (71)$$

the lowest nucleon-pion coupling in the isospin medium is given by $g_{\pi NN} = 11.1675$, which is almost consistent with the experimental data $g_{\pi NN} = 13.1$ [37] like the meson case.

Similarly one can also easily find the interaction terms of the higher resonances in the isospin medium. For example, the second resonances with the even parity, $P_{1440} = \Psi^{(2,+,+)}$ and $N_{1440} = \Psi^{(2,+,-)}$, have the following interaction terms

$$\begin{aligned} S_{int} = i \int d^4x \left[g_{(2,+,+;1,+,+)} \{ \bar{P}(1440) \gamma^5 \pi^0 P + \bar{P} \gamma^5 \pi^0 P_{1440} - \bar{N}(1440) \gamma^5 \pi^0 N - \bar{N} \gamma^5 \pi^0 N_{1440} \right. \\ \left. + \bar{P}_{1440} \gamma^5 \pi^+ N + \bar{P} \gamma^5 \pi^+ N_{1440} + \bar{N}_{1440} \gamma^5 \pi^- P + \bar{N} \gamma^5 \pi^- P_{1440} \} \right. \\ \left. + g_{(2,+,+;2,+,+)} \{ \bar{P}_{1440} \gamma^5 \pi^0 P_{1440} - \bar{N}_{1440} \gamma^5 \pi^0 N_{1440} + \bar{P}_{1440} \gamma^5 \pi^+ N_{1440} \right. \\ \left. + \bar{N}_{1440} \gamma^5 \pi^- P_{1440} \} + \dots \right] . \quad (72) \end{aligned}$$

Like the meson and nucleon masses, the nucleon-pion couplings of the higher resonances also have a large deviation from data which seems to be inevitable in the simple hard wall model.

4 Discussion

In this paper, we have investigated the nucleon spectra and nucleon-pion coupling in the isospin medium which can provide a good playground to study and understand the medium effect due to its simplicity and analyticity. Here we considered a hard wall model with a negative cosmological constant and constant gauge potentials corresponding to the isospin chemical potentials. In general, since the equations of motion do not depend on the gauge potential but on the gauge field strength, the bulk constant gauge potential does not modify the background geometry. However, since bulk fermions are coupled to the gauge potential, some physical quantities associated with fermion can be affected by the constant gauge potentials. This is the story of the gravity theory. Following the AdS/CFT correspondence, the 5-dimensional gravity theory can be reinterpreted as physics of the 4-dimensional dual gauge theory.

In the dual field theory, the constant gauge potentials are mapped to the isospin chemical potentials and the bulk massive fermions which have a mass, either $5/2$ or $-5/2$, are dual to nucleons with a conformal dimension $9/2$. If there is no nontrivial gauge potential, proton and neutron are indistinguishable, so they have the same physical properties like mass spectrum and nucleon-pion coupling. In the isospin medium proton and neutron interact with the isospin matter in a different way due to their different isospin charges so that the nucleon mass splitting occurs. More precisely, the nucleon mass splitting is exactly half of the meson mass splitting because the nucleon isospin charge is half of charged mesons. As expected, the masses of mesons and nucleons increase or decrease linearly as $\mu_P - \mu_N$ increases. In spite of the mass shifts of nucleons and mesons in the isospin medium, we find that there is no the nucleon-pion coupling splitting because the mode functions are independent of the isospin chemical potential of the isospin medium. It would be interesting to apply holographic techniques studied in this paper to the nuclear medium for understanding the more real physical systems like the nuclear matter and neutron star.

Although the hard wall model leads to almost consistent lowest mass spectra with the phenomenological data, the high resonance spectra are quite different from the real data. So, it would be interesting to improve the hard wall model to explain the mass spectra of high excitations well.

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